

1. Lesson Plan Information	
Subject/Course: MAP 4C College and Apprenticeship Mathematics	Name: Adapted from Stephanie Mills
Grade Level: 12	Date: October 17
Topic: The Cosine Law	Time and Length of Period: 9:00-10:14am (74 minutes)

2. Expectation(s) and Learning Skills
<p>The students will:</p> <ul style="list-style-type: none"> Determine side lengths and angle measures in oblique triangles, using the cosine law Solve related problems using the cosine law Solve problems involving trigonometry in right triangles <p>Today, students will:</p> <ul style="list-style-type: none"> Solve related problems using the cosine law

3. Pre-assessment
<p>A. (i) Students</p> <ul style="list-style-type: none"> Students have an understanding of the sine law Students can solve introductory problems involving trigonometry <p>(ii) Differentiation of content, process, and/or product (may be accommodations and/or modifications)</p> <ul style="list-style-type: none"> M may be sent to the resource room for support after the lesson depending on how well he understands the material covered in class
<p>B. Learning Environment</p> <ul style="list-style-type: none"> 4 rows of 2 desks (32 desks in total) Students will work with a partner of their own choosing
<p>C. Resources/Materials</p> <ul style="list-style-type: none"> Textbooks (students have their own) Cosine Law Handout (one for each student) Cosine Law Handout Transparency Overhead projector Overhead markers Markers, chart paper Students' math books

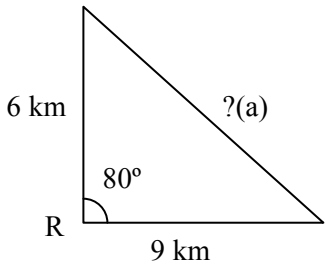
4. Content (The What)	Teaching/Learning Strategies (The How)
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A. Introduction (motivational steps/hook/activation of students' prior knowledge)
 (5 min.)

- Find a partner and on the chart paper given, try to solve the problem that is on the overhead.
- *Example 1:* A radar station is tracking two ships. At midnight, the ships are 6 km and 9 km away from the radar station. The angle between the lines of sight to the ships is 80°. How far apart are the ships at midnight?
- Use a diagram to help you solve the problem. Label the given information on your diagram as well as labelling the unknown side we are trying to find (a). Ask students how to set up the equation using Sine Law $\left\{ \frac{a}{\sin 80^\circ} = \frac{6}{\sin B} = \frac{9}{\sin C} \right\}$.

B. Content for New Learning	B. Teaching/Learning Strategies for New Learning
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- Some triangles cannot be solved using Sine Law alone.



1. Cosine Law is the rule to calculate measures in oblique triangles

To use the Cosine Law, we must know:

- the measures of two sides and their included angle (SAS), or
- the measures of all three sides (SSS)

(45 min.)

As a class, discuss, “What problems arise when trying to use the Sine Law?”

Explain to students that no matter which pair of ratios we choose to work with, there are always two unknown measures. So, Sine Law cannot be used to solve this problem.

There is another law, called the Cosine Law, which we can use to calculate measures in triangles.

1. Teacher demonstrates with the following:
 In any ΔABC :
 $a^2 = b^2 + c^2 - 2bc \cos A$
 $b^2 = a^2 + c^2 - 2ac \cos B$
 $c^2 = a^2 + b^2 - 2ab \cos C$

To use the Cosine Law, we must know:

- the measures of two sides and their included angle (SAS)
- or
- the measures of all three sides (SSS)

Teacher models Example 1.
 Return to opening example. Work out the example on the board with student assistance:
 $a^2 = b^2 + c^2 - 2bc \cos A$
 “What is the first step?”

$$a^2 = 6^2 + 9^2 - 2(6)(9) \cos(80^\circ)$$

$$a^2 = 98$$

“How do we calculate the square root for this question?”

$$a = \sqrt{98}$$

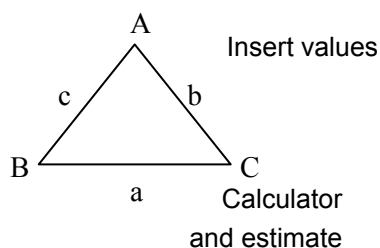
$$a \approx 10$$

∴ The ships are 10km apart at midnight.

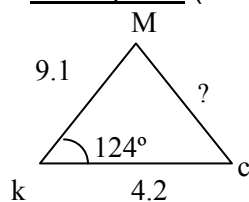
2. Example 1: (SAS)

Formula for the Cosine Law to find missing sides:

$$a^2 = b^2 + c^2 - 2bc \cos A$$



3. Example 2: (SAS)



3. *Example 2:* In $\triangle KLM$ $m = 4.2$, $l = 9.1$ $\angle K = 124^\circ$. Find k .

Draw diagram on chalkboard and label known and unknown information.

$$k^2 = l^2 + m^2 - 2lm \cos k$$

$$k^2 = 9.1^2 + 4.2^2 - 2(9.1)(4.2) \cos 124^\circ$$

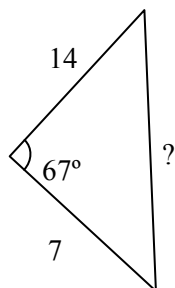
$$k^2 = 143$$

$$k = \sqrt{143}$$

$$k \approx 12.0 \text{ m}$$

Students will work independently to solve example in their notebook.

4. Example 3: (SAS)



4. *Example 3:* A boy walks 14 km south-west, turns 67° toward the south and walks 7 more km. How far is he from his starting point?

Draw diagram on the chalkboard and label known and unknown information.

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$b^2 = 7^2 + 14^2 - 2(7)(14) \cos 67^\circ$$

$$b^2 = 168$$

$$b = \sqrt{168}$$

$$b \approx 13 \text{ km}$$

Formula for using Cosine Law to find missing angles

$$b^2 = a^2 + c^2 - 2ac \cos B$$

Use the law of Sine to calculate any other angle then use the Angle Sum Theorem, (i.e., all angles must add up to 180 degrees)

5. Cosine Law formulae to solve for $\cos A$, $\cos B$, and $\cos C$.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

6. Example 4: (SSS)

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 2bc \cos A = b^2 + c^2$$

$$2bc \cos A = b^2 + c^2 - a^2$$

$$\frac{2bc \cos A}{2bc} = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

7. Example 5: (SSS)

\therefore The boy is 13 km from his starting point.

Walk through the steps on how to solve a triangle using Example 3. What would you do to find another angle? The last angle?

5. Sometimes we know the lengths of all three sides of a triangle, and we need to determine the measure of an angle. Recall, the cosine of an acute angle is positive, and the cosine of an obtuse angle is negative.

On your own, show how the Cosine Law formulae can be arranged in order to solve for $\cos A$.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 + 2bc \cos A = b^2 + c^2$$

$$2bc \cos A = b^2 + c^2 - a^2$$

$$\frac{2bc \cos A}{2bc} = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

6. *Example 4:* In $\triangle ABC$, $a = 8.3$ cm, $b = 4.7$ cm, and $c = 6.1$ cm. Determine the measure of angle A to the nearest degree.

Draw a diagram on the chalkboard and label known and unknown information.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{4.7^2 + 6.1^2 - 8.3^2}{2(4.7)(6.1)}$$

$$\cos A = -0.167$$

$$\cos^{-1}(\cos A) = \cos^{-1}(-0.167)$$

$$A = \cos^{-1}(-0.167)$$

$$A = 99.6$$

$$\therefore A \approx 100^\circ$$

Angle A would measure approximately 100° .

7. *Example 5:* A foosball net is 48 cm wide. A player is 75 cm from one goal post and 80 cm from the other. Within what angle must he keep his shot in order to score a goal? Work in pairs to complete this question.

Draw a diagram and label the known and unknown information.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{80^2 + 75^2 - 48^2}{2(80)(75)}$$

$$\cos A = 0.810$$

$$\cos^{-1}(\cos A) = \cos^{-1}(0.810)$$

$$A = \cos^{-1}(0.810)$$

$$A = 35.8$$

$$\therefore A \approx 36^\circ$$

5. Consolidation/Recapitulation Questions (Check for understanding / scaffolded practice)

- Ask the students: “Under which conditions can Cosine Law be used?”
- “What information do we need to know?” Answer: SAS or SSS
- Provide two or three additional practice examples and close supervision to ensure that students understand each before starting work from their texts independently.

6. Application (Moving from guided, scaffolded practice to increasingly independent practice and understanding / gradual release of responsibility)

(20 min.)

- Have students fill in the Cosine Law Handout with the teacher filling in the Overhead copy as answers are being stated.
1. Why were we not able to solve the first example with the Law of Sine?
 2. What is the formula for the Law of Cosine?
 3. Under what conditions do you use the Law of Cosine?
 4. Does it matter if the angle between 2 sides of the triangle is acute or obtuse? Does the Law of Cosine apply?
 5. In solving triangles, what is the procedure you should always follow?
 6. Students should complete the following exercises to apply these principles:
 - Pg. #47 Q. # 1, 4, 6a-c, 7, 9, 10, 11, 13
 - Pg. #48 Q. #3, 5, 6b-d, 8, 12, 15

7. Lesson Conclusion

(2-3 min.)

- Ask the students what they learned about when they would use the Cosine Law.

8. Assessment (collection of data) / Evaluation (interpretation of data)

- Questions from page 48 numbers 12 and 15 to be handed in for marking in tomorrow’s class.
- One mark will be given for the diagram, one for choosing the appropriate law, one substitution correctly and one for the final answer for a total of 4 marks in the Application category.
- Evaluations will be recorded on a class checklist.

9. Student Teachers' Reflections on the Lesson

A. (i) Evidence of Student Learning Related to Lesson Expectation(s)

- Students grasped how to determine side lengths and angle measures in oblique triangles, using the cosine law, and solved related problem. They were having some difficulty solving trigonometry independently.
- M did receive additional support in the resource room.
- All students were able to identify under which conditions Cosine Law can be used.
- M was able to complete the assignment with the additional support in the resource room

(ii) Next Steps for Student Learning Related to Lesson Expectation(s)

- Tomorrow start off class with some review questions. Assign students to work at the board. Assign other students to make corrections and present answers to the class.
- I will continue to monitor M.

B. (i) Evidence of the Effectiveness of the Student Teacher

- The examples were clear and concise. Students were able to follow along without difficulty. More student participation when answering questions is needed – only a couple of students answer questions. Some students were off task and talking during the application.
- The data gathered using the checklist, indicated that all students were able to meet the expectations for the lesson.

(ii) Next Steps Related to the Effectiveness of the Student Teacher

- I will be sure to pre-plan all my recapitulation questions for further lessons.